



GWINEAR SCHOOL

CALCULATION POLICY

Written: April 2019

Reviewed: June 21, June 24

Written by: Maths Coordinator in consultation with Maths Hub

Agreed with Maths Governor: Charlotte Willson

Review Date: June 27

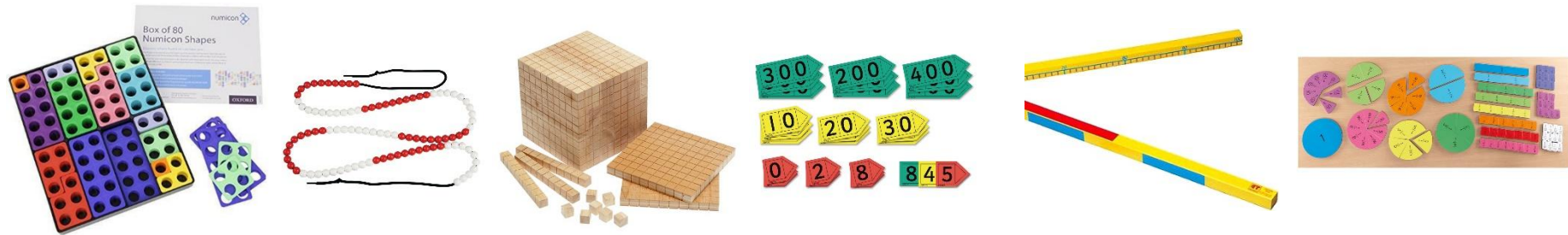


This policy has been adapted from the White Rose Maths Hub Calculation Policy with further material added including videos demonstrating each process. It is a working document and will be revised and amended as necessary.

The overall aims of this policy are that, when children leave primary school they:

- have a secure knowledge of number facts and a good understanding of the four operations supported by a fluency and understanding of the fundamentals of mathematics
- Know the best strategy to use, estimate before calculating, systematically break problems down into a series of simpler steps with perseverance and use estimation and rounding to check that an answer is reasonable
- Are able to use this knowledge and understanding to carry out calculations mentally, solve problems of increasing complexity and develop an ability to recall and apply knowledge rapidly.
- Make use of diagrams and informal notes and jottings to help record steps and partial answers when using mental methods
- Have an efficient, reliable, compact written method of calculation for each operation, which they can apply with confidence when undertaking calculations
- Be able to identify when a calculator is the best tool for the task and use this primarily as a way of checking rather than simply a way of calculating.
- Be able to explain their strategies to calculate and, using spoken language, give mathematical justification, argument or proof.

Manipulatives used to aid/teach mathematics



Numicon	Bead strings	Base 10	Place value cards	Counting stick	Fraction/decimal tiles
<ul style="list-style-type: none"> odd and even arrays division multiplication Number bonds 	<ul style="list-style-type: none"> number bonds counting in tens tenths 	<ul style="list-style-type: none"> place value addition subtraction 	<ul style="list-style-type: none"> understanding of place value addition subtraction 	<ul style="list-style-type: none"> counting in regular integers measures 	<ul style="list-style-type: none"> Understanding equivalent fractions Addition and subtraction of fractions Understanding decimal-fraction equivalents

Year Group	Numicon	Counters	Bead strings	Base 10	Place value chart/cards	Number lines	Counting sticks	Fraction tiles	Decimal tiles	Money
R	✓	✓	✓			✓	✓			✓
1	✓	✓	✓		✓	✓	✓			✓
2	✓	✓	✓	✓	✓	✓	✓	✓		✓
3	✓	✓	✓	✓	✓	✓	✓	✓		✓
4	✓	✓	✓	✓	✓	✓	✓	✓	✓	
5	✓	✓		✓	✓	✓	✓	✓	✓	
6	✓	✓		✓	✓	✓	✓	✓	✓	

Term by term objective

This is a guide to show how much time should be spent on block units of learning. The majority of learning should be taught as a block unit. These time timetables are flexible can vary depending on needs of children and assessment periods.

Number	Measures	Geometry	Statistics	Consolidation
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Class 1: Reception

Children learn through play and at their own pace. Teachers will assess when children are ready to tackle the two math areas to work towards achieving the early learning goals (number and shape space and measures)

Class 2: Year 1 and 2

	1	2	3	4	5	6	7	8	9	10	11	12
Autumn	Place value						Addition & subtraction				Multiplication & Division	
Spring	Fractions				Time		Shape		Length/height/mass		Consolidation	
Summer	Place value/Money			Statistics	Consolidation		Place value				Consolidation	

Class 3: Year 3 and 4

	1	2	3	4	5	6	7	8	9	10	11	12
Autumn	Place value			Addition and Subtraction			Multiplication and Division				Measures	
Spring	Fractions					Time	Decimals				Statistics	
Summer	Shape/symmetry/position/direction/angles					Consolidation	Length/perimeter/area				Consolidation	

Class 4: Year 5 and 6

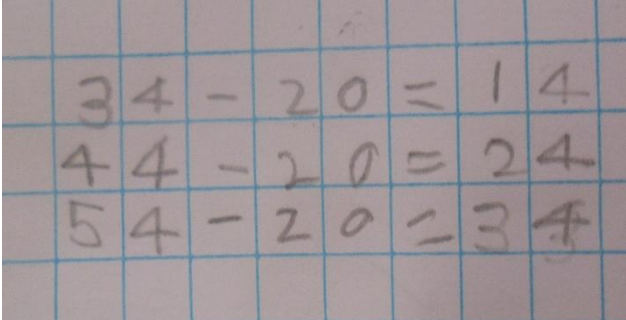
	1	2	3	4	5	6	7	8	9	10	11	12
Autumn	Place Value			Addition/subtraction/multiplication/division					Fractions			
Spring	Decimals/percentages/ratio						Converting units		Perimeter/area/volume		angles	Consolidation
Summer	Properties/position & direction			Algebra	Statistics		prime	Consolidation				

Presentation and teaching

EYFS: Recording will be completed through observations.

KS1:

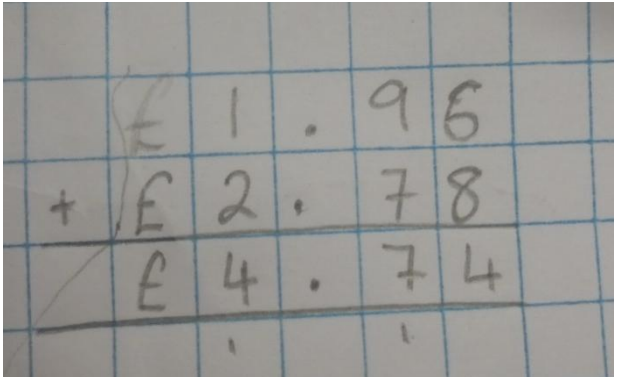
- All numerals should be written a digit per box
- Symbols should also be written in their own box
- Children should use math books that have 1cm squares



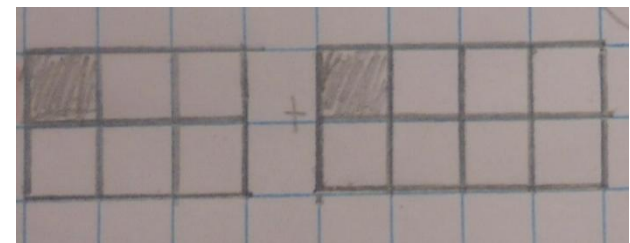
Three subtraction problems are written on 1cm grid paper. Each digit and symbol is written in its own box. The problems are: $34 - 20 = 14$, $44 - 20 = 24$, and $54 - 20 = 34$.

KS2:


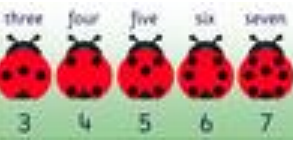

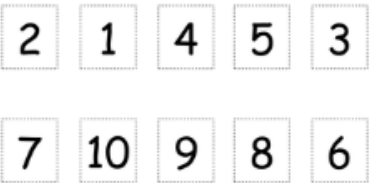

- All numerals must be written a digit per box
- Symbols and decimal points must be written in their own box
- Children should use math books that have 8mm squares
- All diagrams and lines for fractions must be drawn on the lines in the book with a ruler



A handwritten addition problem is shown on 8mm grid paper. The problem is $£1.96 + £2.78 = £4.74$. The pounds symbol (£) and decimal point (.) are written in their own boxes. A horizontal line is drawn under the second number.



Counting: Reception

Objective & Strategy	Concrete	Pictorial	Abstract
Count on from any number to 20	Use dominos/numicon to ask children to continue the sequence 	Show children images of amounts e.g. ladybirds with different spots and ask children what comes next. 	Ask children to continue counting allowed after you finish "5, 6, 7..." "13, 14, 15..."
Order numbers to 20	Ask children to arrange numicon or groups of objects into the correct order	Mix up picture cards and ask children to reorder the cards 	*Ask children to physical order numbers to 20 by moving pictures of numerals 
*Estimate a number of objects and check by counting to 20	Children guess the amount of objects in a group before counting using 1:1 correspondence 	N/A	N/A


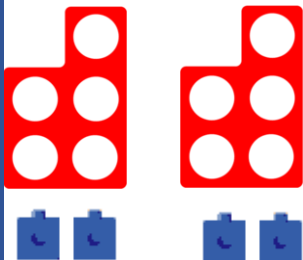
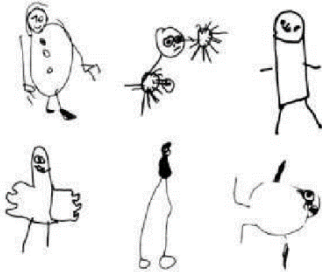
Addition

Language to be used

This is a progression of language and shows when new language should be introduced, language should still be used throughout the years once introduced to children.

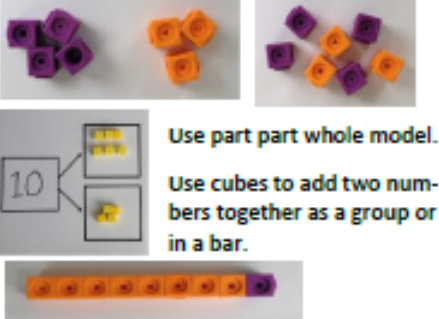
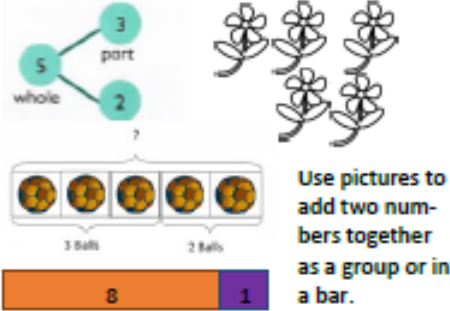
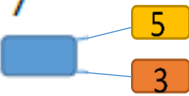

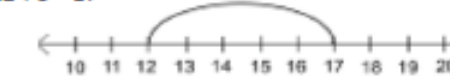
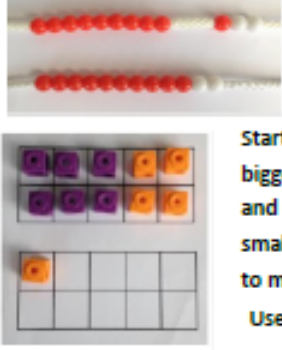

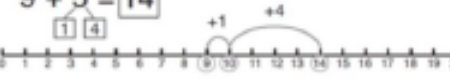

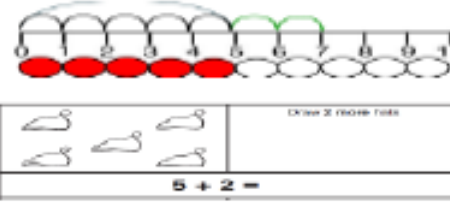
Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<ul style="list-style-type: none"> • add • more • and make • sum • total • altogether • score • double one more, two more, ten more... • how many more to make... ? • how many more is... than...? 	<ul style="list-style-type: none"> • plus • how much more is...? 	<ul style="list-style-type: none"> • addition • one hundred more • tens boundary • amount • inverse 	<ul style="list-style-type: none"> • hundreds boundary • calculator 	<ul style="list-style-type: none"> • increase • unit boundary • currency 	<ul style="list-style-type: none"> • units boundary • tenths boundary • brackets 	<ul style="list-style-type: none"> • commutative • complements (10,100) • exact/exactly • most significant digit

Addition: Reception

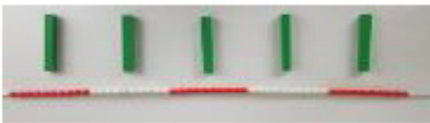



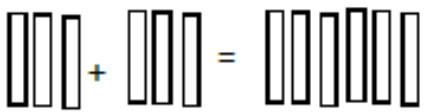
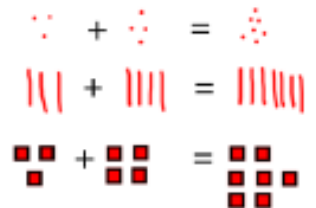


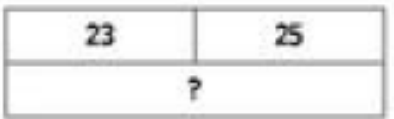
Objective & Strategy	Concrete	Pictorial	Abstract
Add two single digit numbers and count/count on to find the answer	<p>Show children two groups of objects and ask them how many there are altogether. Children should use 1:1 correspondence to count objects. “How much fruit do we have?”</p> 	<p>Show children pictures of objects/dots and ask children how many there are altogether.</p>	<p>Children may begin to say number sentence aloud using “add or and” “2 and 3 is 5” Progress to meet year 1 objective</p>
*Solve practical problems by combining groups of 2, 5 and 10	<p>Children shown/given objects/numicon in groups of 2, 5 and 10 and asked to add them together without counting the amounts in both groups</p> 	<p>Children to count pictures of dots or other objects on a picture or drawing (independent drawing or provided) and say amount aloud *or write numeral</p> 	N/A

Addition: Year 1

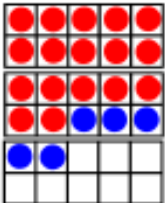
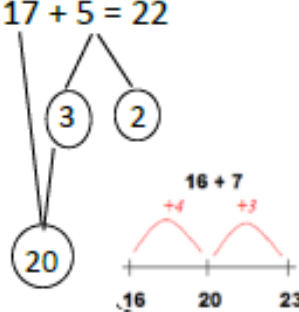

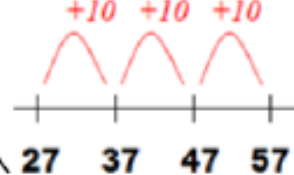

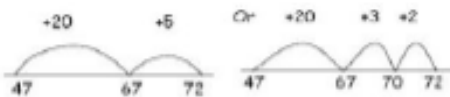
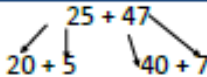


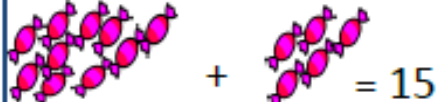


Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	$4 + 3 = 7$  <p>Use the part-part whole diagram as shown above to move into the abstract.</p> $10 = 6 + 4$
Starting at the bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	$12 + 5 = 17$  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
Regrouping to make 10. <i>This is an essential skill for column addition later.</i>	 <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames.</p> $6 + 5 = 11$	 <p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.</p> $9 + 5 = 14$ 	$7 + 4 = 11$ <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>
Represent & use number bonds and related subtraction facts within 20	 <p>2 more than 5.</p>	 <p>5 + 2 =</p>	<p>Emphasis should be on the language</p> <p>'1 more than 5 is equal to 6.'</p> <p>'2 more than 5 is 7.'</p> <p>'8 is 3 more than 5.'</p>

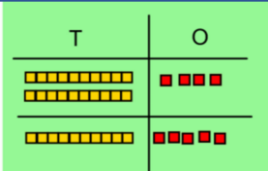
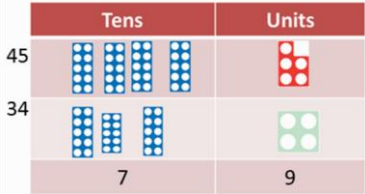
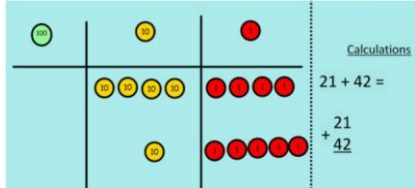
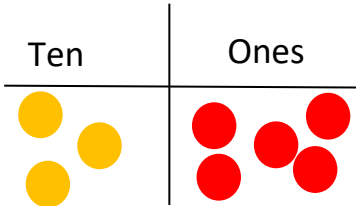
Addition: Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \text{ } \text{tens}$ $30 + 50 = \text{ }$ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts <i>Part part whole</i>	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts	$\square\square + \square\square = \square\square\square\square$ 	 Children draw representations of H, T and O	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$



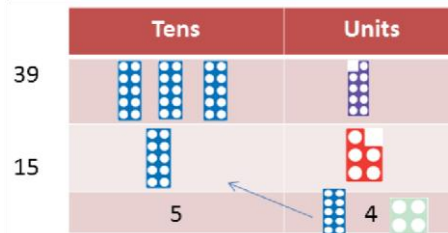
Objective & Strategy	Concrete	Pictorial	Abstract				
Add a two digit number and ones	<div></div> <div>$17 + 5 = 22$</div> <div>Use ten frame to make 'magic ten'</div> <div>Children explore the pattern.</div> <div>$17 + 5 = 22$</div> <div>$27 + 5 = 32$</div>	<div>$17 + 5 = 22$</div> <div>Use part part whole and number line to model.</div> <div></div>	<div>$17 + 5 = 22$</div> <div>Explore related facts</div> <div>$17 + 5 = 22$</div> <div>$5 + 17 = 22$</div> <div>$22 - 17 = 5$</div> <div>$22 - 5 = 17$</div> <div><table border="1" data-bbox="1796 528 2020 608"><tr><td colspan="2">22</td></tr><tr><td>17</td><td>5</td></tr></table></div>	22		17	5
22							
17	5						
Add a 2 digit number and tens	<div></div> <div>$25 + 10 = 35$</div> <div>Explore that the ones digit does not change</div>	<div>$27 + 30$</div> <div></div>	<div>$27 + 10 = 37$</div> <div>$27 + 20 = 47$</div> <div>$27 + \square = 57$</div>				
Add two 2-digit numbers	<div></div> <div>Model using dienes, place value counters and numicon</div>	<div></div> <div>Use number line and bridge ten using part whole if necessary.</div>	<div>$25 + 47$</div> <div></div> <div>$20 + 40 = 60$</div> <div>$5 + 7 = 12$</div> <div>$60 + 12 = 72$</div>				
Add three 1-digit numbers	<div></div> <div>Combine to make 10 first if possible, or bridge 10 then add third digit</div>	<div></div> <div>Regroup and draw representation.</div> <div></div> <div>$= 15$</div>	<div>$4 + 7 + 6 = 10 + 7$</div> <div>$= 17$</div> <div>Combine the two numbers that make/ bridge ten then add on the third.</div>				

Addition: Year 3

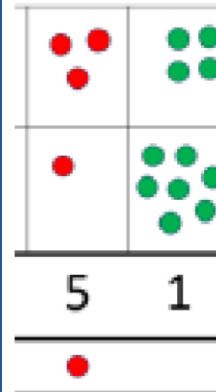
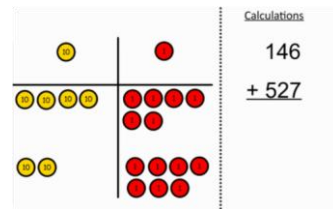
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3digit numbers.</p>	<p>Model using Dienes or numicon</p>  <p>Add together the ones first, then the tens.</p>   <p>Move to using place value counters</p>	<p>Children move to drawing the counters using a tens and one frame.</p> 	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>



Column Addition
with regrouping.



Exchange ten ones for a ten. Model
using numicon and pv counters.



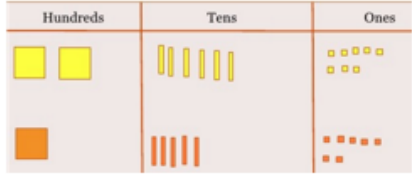
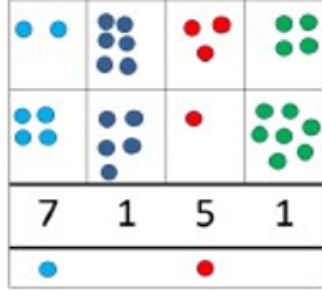
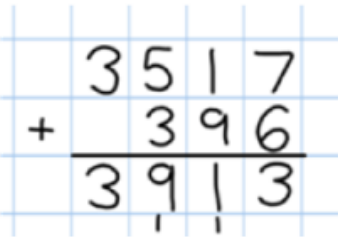
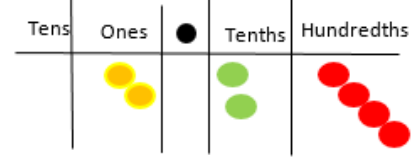
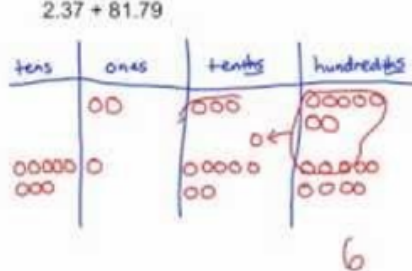
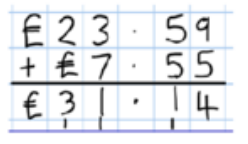
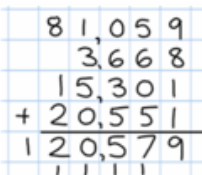
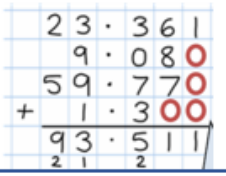
Children can
draw a
representation
of the grid to
further support
their
understanding,
carrying the ten
underneath the
line

Start by partitioning
the numbers before
formal column to show
the exchange.

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

Addition: Year 4/5/6



Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	<p>Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Draw representations using pv grid.</p>	 <p>Continue from previous work to carry hundreds as well as tens. Relate to money and measures.</p>
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	<p>As year 4</p>  <p>Introduce decimal place value counters and model exchange for addition.</p>	<p>2.37 + 81.79</p> 	<p>72.8 + 54.6 127.4</p> <p>11</p> 
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	 <p>Insert zeros for place holders.</p> 



Subtraction

Language to be used

This is a progression of language and shows when new language should be introduced, language should still be used throughout the years once introduced to the children.

Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<ul style="list-style-type: none"> • take (away) • leave • how many are left/left over? • how many have gone? • one less, two less... ten less... • how many fewer is... than...? • difference between is the same as 	<ul style="list-style-type: none"> • subtract • minus • how much less is...? • half/halve 	<ul style="list-style-type: none"> • one hundred less • tens boundary • inverse 	<ul style="list-style-type: none"> • hundreds boundary 	<ul style="list-style-type: none"> • decrease • change 	<ul style="list-style-type: none"> • units boundary • tenths boundary 	<ul style="list-style-type: none"> • least significant digit • discount

Subtraction: Reception

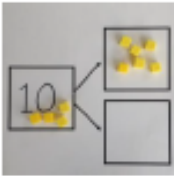
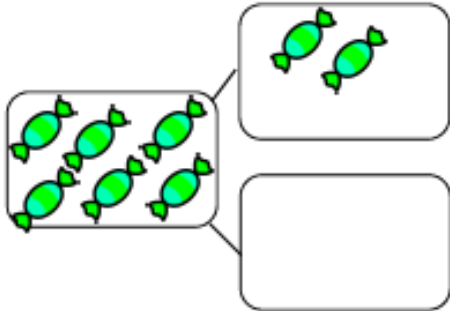





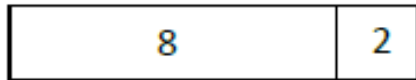
Objective & Strategy	Concrete	Pictorial	Abstract
Subtract two single digit numbers by counting remainder or counting back	<p>Show children a group of objects and take some away. Children should be able to say how many are left</p> 	<p>Show children a picture and ask how many there would be if I took (single digit number) away "How many would I have if you took 1 balloon"</p> 	<p>Progress to meet year 1 objective</p>

Subtraction: Year 1



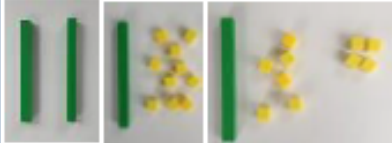
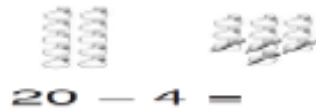


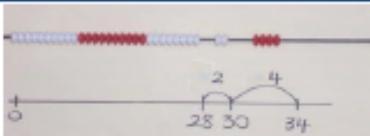
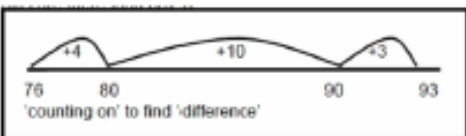
Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p> <p>$6 - 4 = 2$</p> <p>$4 - 2 = 2$</p>	<p>$15 - 3 = 12$</p> <p>Cross out drawn objects to show what has been taken away.</p>	<p>$7 - 4 = 3$</p> <p>$16 - 9 = 7$</p>
Counting back	<p>Move objects away from the group, counting backwards.</p> <p>Move the beads along the bead string as you count backwards.</p>	<p>$5 - 3 = 2$</p> <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p>
Find the Difference	<p>Compare objects and amounts</p> <p>7 'Seven is 3 more than four'</p> <p>4 'I am 2 years older than my sister'</p> <p>3 Pencils</p> <p>3 Erasers</p> <p>7</p> <p>Lay objects to represent bar model.</p>	<p>Count on using a number line to find the difference.</p> <p>$+6$</p>	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?</p>



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Part Part Whole model</p>	 <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what's the other part?</p> $10 - 6 = 4$	 <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p> 
<p>Make 10</p>	<p>14—9</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p>13—7</p>  <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p>16—8</p> <p>How many do we take off first to get to 10? How many left to take off?</p>
<p>Bar model</p>	 $5 - 2 = 3$		 $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$


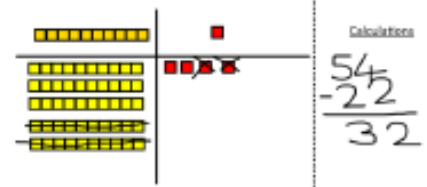
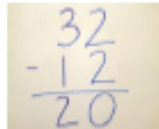
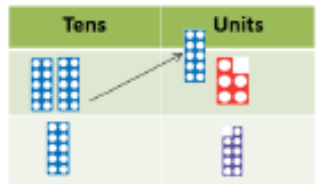
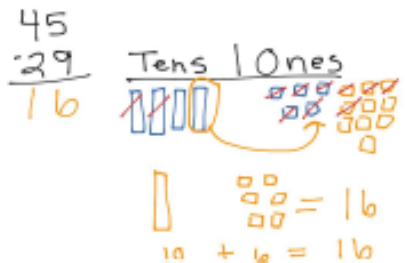
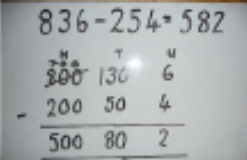
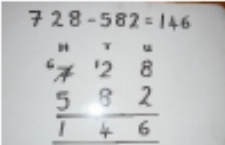
Subtraction: Year 2



Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 $20 - 4 =$	$20 - 4 = 16$
Partitioning to subtract without regrouping. <i>'Friendly numbers'</i>	$34 - 13 = 21$  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  $43 - 21 = 22$	$43 - 21 = 22$
Make ten strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	 $34 - 28$ <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	$93 - 76 = 17$

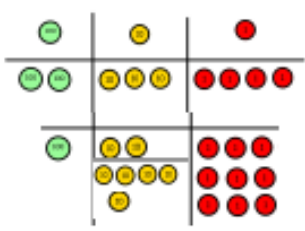
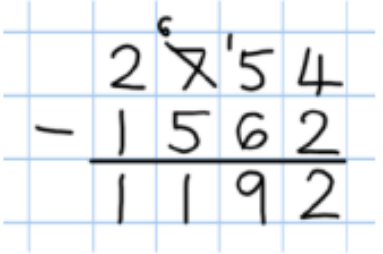
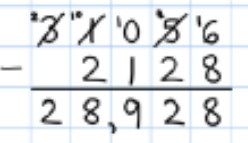
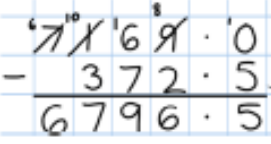
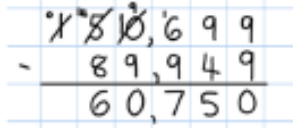
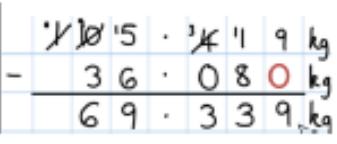
Subtraction: Year 3



Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 <p>47—32</p> <p>Use base 10 or Numicon to model</p>	 <p>Draw representations to support understanding</p>	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 
Column subtraction with regrouping	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p>	 <p>Children may draw base ten or PV counters and cross off.</p>	 <p>Begin by partitioning into pv columns</p>  <p>Then move to formal method.</p>

Subtraction: Year 4/5/6



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Subtracting tens and ones</p> <p>Year 4 subtract with up to 4 digits.</p> <p><i>Introduce decimal subtraction through context of money</i></p>	<p>234 - 179</p>  <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use the phrase 'take and make' for exchange</p>
<p>Year 5- Subtract with at least 4 digits, including money and measures.</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i></p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use zeros for place-holders.</p> 
<p>Year 6—Subtract with increasingly large and more complex numbers and decimal values.</p>			 

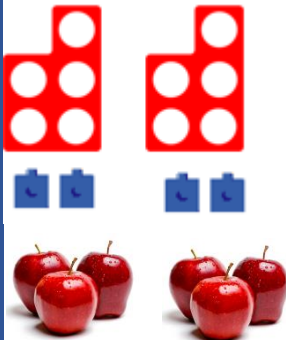
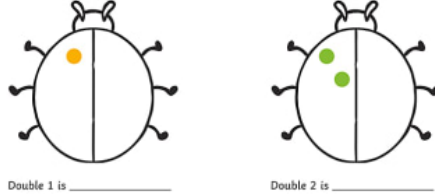
Multiplication

Language to be used

This is a progression of language and shows when new language should be introduced, language should still be used throughout the years once introduced to the children.

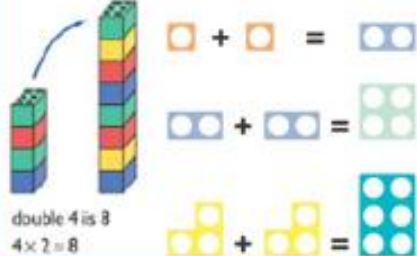

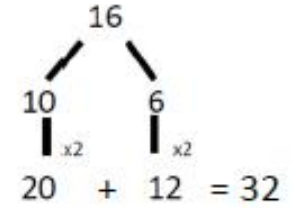

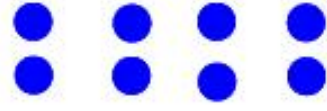




Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<ul style="list-style-type: none"> double 	<ul style="list-style-type: none"> lots of times multiply multiplied once, twice... big, long, wide repeated addition pairs 	<ul style="list-style-type: none"> multiple array column row inverse 	<ul style="list-style-type: none"> multiplication product 	<ul style="list-style-type: none"> factor equivalent quotient 	<ul style="list-style-type: none"> short multiplication long multiplication 	

Multiplication: Reception

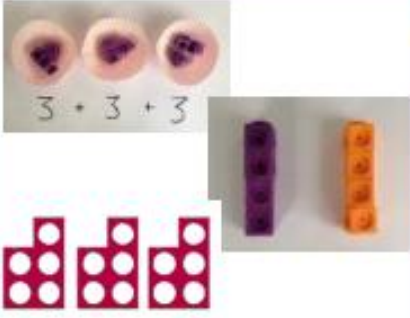
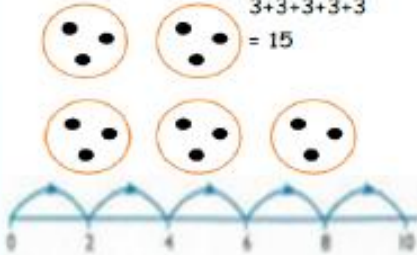

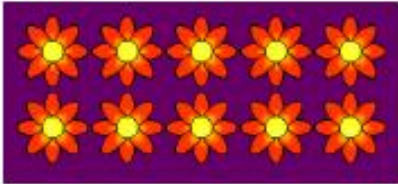
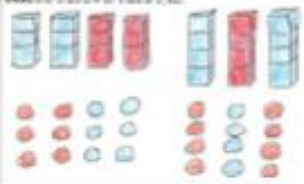
Objective & Strategy	Concrete	Pictorial	Abstract
Doubling numbers to 10	<p>Children can count two groups of objects of the same amount and understand that this is doubling</p> 	<p>Children can copy amount of dots or images to show the doubling is a number multiplied by 2 and count final amount by using 1:1 correspondence</p> 	<p>"Double 3 is 6"</p>

Multiplication: Year 1



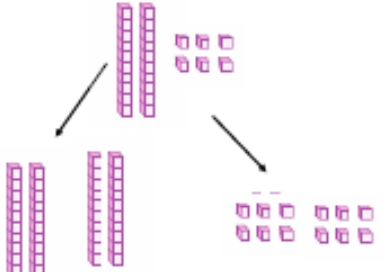
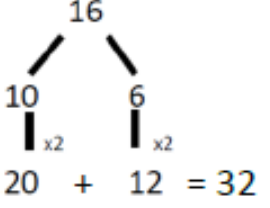


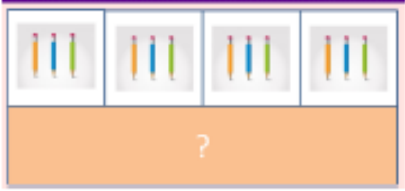



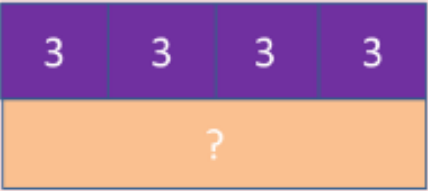
Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p>  <p>16 10 and 6 $10 \times 2 = 20$ $6 \times 2 = 12$ $20 + 12 = 32$</p>
Counting in multiples	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>
Making equal groups and counting the total	  <p>$\square \times \square = 8$</p> <p>Use manipulatives to create equal groups.</p>	<p>Draw  to show $2 \times 3 = 6$</p> <p>Draw and make representations</p>	<p>$2 \times 4 = 8$</p>

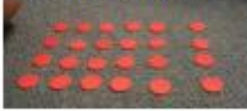




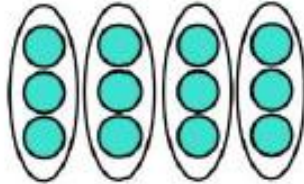
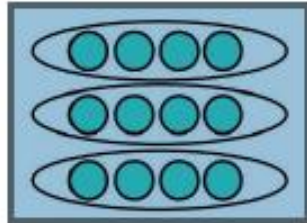


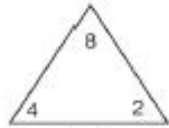


Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve prob</p> <p>There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p>  <p>$3+3+3+3+3 = 15$</p>	<p>Write addition sentences to describe objects and pictures.</p>  <p>$2+2+2+2+2 = 10$</p>
Understanding arrays	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding.</p> 	<p>$3 \times 2 = 6$</p> <p>$2 \times 5 = 10$</p>

Multiplication: Year 2



Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Model doubling using dienes and PV counters.</p>  $40 + 12 = 52$	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p>  $20 + 12 = 32$
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p>  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$  	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p>    	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10</p> <p>0, 3, 6, 9, 12, 15</p> <p>0, 5, 10, 15, 20, 25, 30</p> $4 \times 3 = \square$

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters and cubes and Numicon.</p>    <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p>  	<p>Use representations of arrays to show different calculations and explore commutativity.</p>  	<p>$12 = 3 \times 4$</p> <p>$12 = 4 \times 3$</p> <div> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$</p> <p>$3 + 3 + 3 + 3 + 3 = 15$</p> <p>$5 \times 3 = 15$</p> <p>$3 \times 5 = 15$</p> </div>
<p>Using the Inverse</p> <p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p>		 <div> $\square \times \square = \square$ $\square \times \square = \square$ $\square \div \square = \square$ $\square \div \square = \square$ </div>	<p>$2 \times 4 = 8$</p> <p>$4 \times 2 = 8$</p> <p>$8 \div 2 = 4$</p> <p>$8 \div 4 = 2$</p> <p>$8 = 2 \times 4$</p> <p>$8 = 4 \times 2$</p> <p>$2 = 8 \div 4$</p> <p>$4 = 8 \div 2$</p> <p>Show all 8 related fact family sentences.</p>



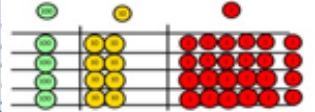

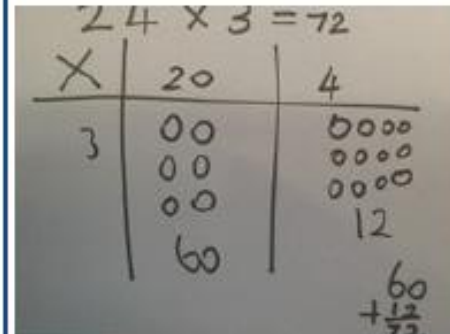
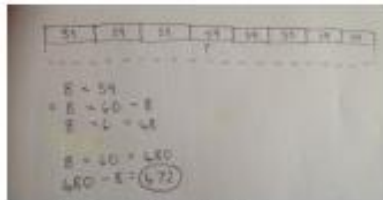
Multiplication: Year 3



Objective & Strategy	Concrete	Pictorial	Abstract															
Grid method	<p>Show the links with arrays to first introduce the grid method</p> <div><div><div><div>x</div><div>10</div><div>3</div></div><div><div>4</div><div></div></div></div><div>4 rows of 10 4 rows of 3</div></div> <p>Move onto base ten to move towards a more compact method.</p> <div><div><div><div>x</div><div>T</div><div>U</div></div><div><div></div></div></div><div>4 rows of 13</div></div> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p> <div><div><div><div></div><div></div><div></div></div><div></div></div><div>Calculations 4 x 126</div></div> <p>Fill each row with 126</p> <div><div><div><div></div><div></div><div></div></div><div></div></div><div>Calculations 4 x 126</div></div> <p>Add up each column, starting with the ones making any exchanges needed</p> <div><div><div><div></div><div></div><div></div></div><div></div></div><div><div><div><div></div><div></div><div></div></div><div></div></div></div><p>Then you have your answer.</p></div> <td><p>Children can represent their work with place value counters in a way that they understand.</p><p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p><div><div><div>24 x 3 = 72</div><div><div><div>X</div><div>20</div><div>4</div></div><div><div>3</div><div><div>00</div><div>00</div><div>00</div><div>60</div></div><div><div>0000</div><div>0000</div><div>0000</div><div>12</div><div>60</div><div>+ 12</div><div>72</div></div></div></div></div><p>Bar model are used to explore missing numbers</p><div><div>4 x <input type="text"/> = 20</div><div><div><div>20</div><div>4</div></div></div></div></div></td> <td><p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p><table><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table><p>210 + 35 = 245</p><p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p><table><tr><td></td><td>10</td><td>8</td></tr><tr><td>10</td><td>100</td><td>80</td></tr><tr><td>3</td><td>30</td><td>24</td></tr></table></td>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> <div><div><div>24 x 3 = 72</div><div><div><div>X</div><div>20</div><div>4</div></div><div><div>3</div><div><div>00</div><div>00</div><div>00</div><div>60</div></div><div><div>0000</div><div>0000</div><div>0000</div><div>12</div><div>60</div><div>+ 12</div><div>72</div></div></div></div></div><p>Bar model are used to explore missing numbers</p><div><div>4 x <input type="text"/> = 20</div><div><div><div>20</div><div>4</div></div></div></div></div>	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> <p>210 + 35 = 245</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table><tr><td></td><td>10</td><td>8</td></tr><tr><td>10</td><td>100</td><td>80</td></tr><tr><td>3</td><td>30</td><td>24</td></tr></table>	x	30	5	7	210	35		10	8	10	100	80	3	30	24
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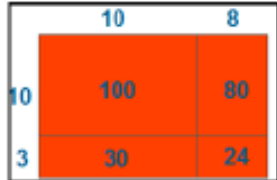
Multiplication: Year 4



Objective & Strategy	Concrete	Pictorial	Abstract																																				
<p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4×126</p> <p>Fill each row with 126</p> <p>Add up each column, starting with the ones</p>  <p>making any exchanges needed</p> <p>Then you have your answer.</p>	<p>Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1709 437 2002 525"><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> <p>$210 + 35 = 245$</p>	x	30	5	7	210	35																														
x	30	5																																					
7	210	35																																					
Column multiplication	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p> <table border="1" data-bbox="703 1002 994 1347"><tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>2</td><td>4</td><td>2</td></tr><tr><td>6</td><td>4</td><td>2</td></tr></table> <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding long multiplication is modelled alongside</p>	Hundreds	Tens	Ones	3	2	1	2	4	2	6	4	2	<table border="1" data-bbox="1238 901 1545 975"><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table> <p>The grid method may be used to show how this relates to a formal written method.</p>  <p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>	x	300	20	7	4	1200	80	28	<p>327</p> <p>x 4</p> <hr/> <p>28</p> <p>80</p> <hr/> <p>1200</p> <hr/> <p>1308</p> <p>This may lead to a compact method.</p> <table border="1" data-bbox="1655 1241 1868 1402"><tr><td>x</td><td>3</td><td>2</td><td>7</td></tr><tr><td>4</td><td></td><td></td><td></td></tr><tr><td></td><td>1</td><td>3</td><td>0</td></tr><tr><td></td><td></td><td>2</td><td>8</td></tr></table>	x	3	2	7	4					1	3	0			2	8
Hundreds	Tens	Ones																																					
3	2	1																																					
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4	1200	80	28																																				
x	3	2	7																																				
4																																							
	1	3	0																																				
		2	8																																				

Multiplication: Year 5/6



Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.	As shown in year 4.		<p>Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.</p> $\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array}$
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 <p>Continue to use bar modelling to support problem solving</p>	<p>18 x 3 on the first row (8 x 3 = 24, carrying the 2 for 20, then 1 x 3)</p> <p>18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first</p> $\begin{array}{r} 1234 \\ \times 16 \\ \hline 7404 \\ 12340 \\ \hline 19744 \end{array}$ <p>(1234 x 6) (1234 x 10)</p>


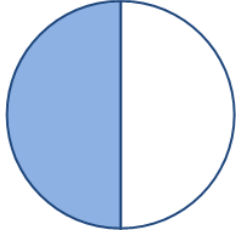
Division

Language to be used

This is a progression of language and shows when new language should be introduced, language should still be used throughout the years once introduced to the children.



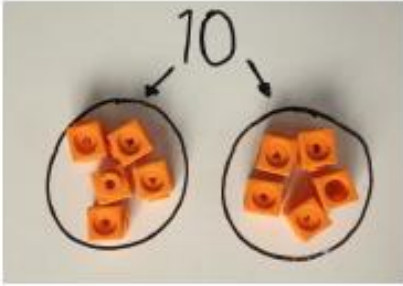

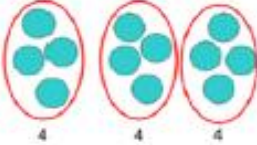
Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<ul style="list-style-type: none"> • half/halve • share 	<ul style="list-style-type: none"> • share equally • one each, two each, three each... • groups of • pairs • divide • divided • left over 	<ul style="list-style-type: none"> • inverse 	<ul style="list-style-type: none"> • division • remainder 	<ul style="list-style-type: none"> • divisible by • factor • quotient • bus stop 	<ul style="list-style-type: none"> • long division • short division 	

Division: Reception

Objective & Strategy	Concrete	Pictorial	Abstract
Halving amounts and objects between 2-10	<p>Dividing an amount of objects between two people "Share these grapes fairly/equally/into two groups"</p> 	<p>Draw a line to show half of a shape</p> 	Half of 6 is 3
Solve problems with sharing	<p>Children can share physical objects equally by giving each group/child one each until they are all gone/used. "Can you share these grapes with your friends?"</p>	N/A	N/A

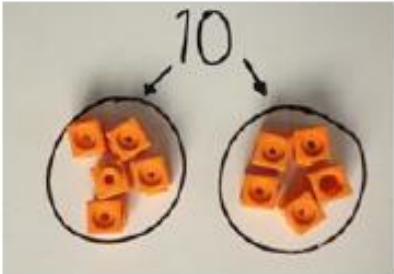

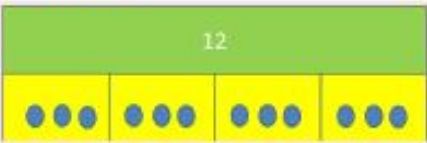


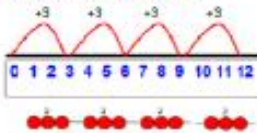

Division: Year 1



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p> <p>Use Gordon ITPs for modelling</p>	   <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>8 8 shared between 2 is 4</p> <p>Sharing:</p>  <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is 4</p> <p>4</p>

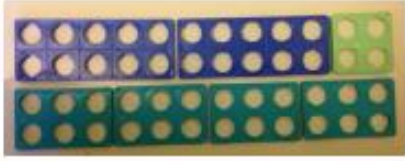
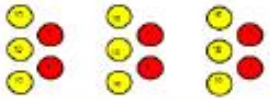


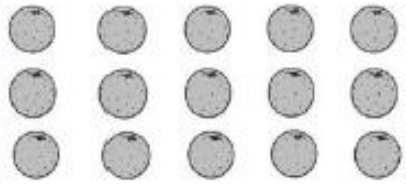
Division: Year 2



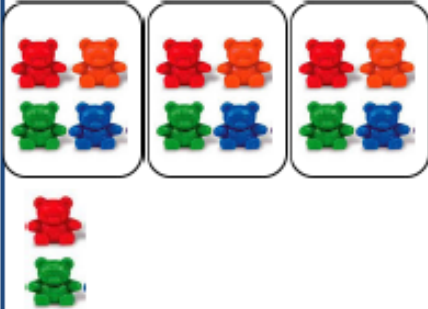


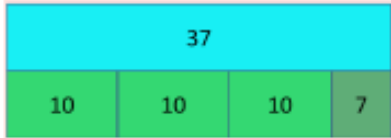
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>$8 \div 2 = 4$</p> <p>Children use bar modelling to show and support understanding.</p>  <p>$12 \div 4 = 3$</p>	$12 \div 3 = 4$
Division as grouping	<p>Divide quantities into equal groups.</p> <p>Use cubes, counters, objects or place value counters to aid understanding.</p>  	<p>Use number lines for grouping</p>  <p>$12 \div 3 = 4$</p> <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p>$20 \div 5 = ?$ $5 \times ? = 20$</p>	$28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p>

Division: Year 3



Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
Division with arrays	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

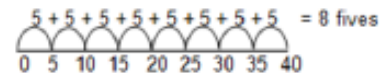


Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	<p>$14 \div 3 =$</p> <p>Divide objects between groups and see how much is left over</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p> 	<p>Complete written divisions and show the remainder using r.</p> <p>$29 \div 8 = 3 \text{ REMAINDER } 5$</p> <p>↑ ↑ ↑ ↑</p> <p>dividend divisor quotient remainder</p>

Example without remainder:

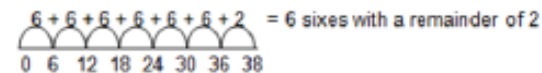
$40 \div 5$

Ask "How many 5s in 40?"



Example with remainder:

$38 \div 6$



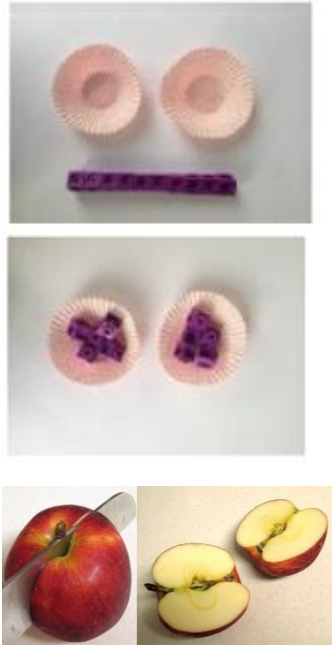

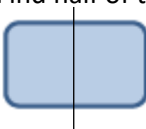
For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.

Division: Year 4-6


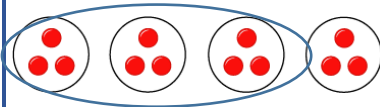

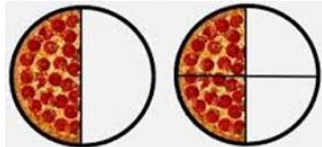
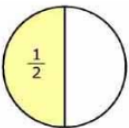
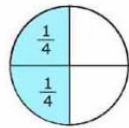


Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	<p>$96 \div 3$</p> <p>Tens Units</p> <p>3 2</p> <p>Use place value counters to divide using the bus stop method alongside</p> <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p> <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p> <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p> <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$

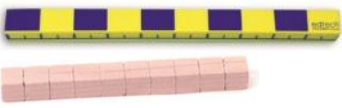


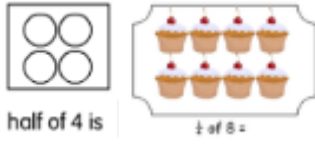

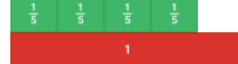
Fractions: Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Recognise, find and name a half and a quarter of an object shape or quantity.</p> <p>Fractions through division</p>	<p>Use objects and groups of objects to show half or a quarter of a whole as dividing by 2 or 4.</p> 	<p>Use pictorial representations to show dividing the whole group by 2 (for halves or 4 for quarters)</p> 	<p>Find $\frac{1}{2}$ of 8</p> <p>$\frac{1}{2}$ of 8 = 4</p> <p>Find half of this shape</p>  <p>Use a ruler to divide a shape into quarters or halves. Start with a rectangle/square and move onto more complex shapes.</p>





Fractions: Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Recognise, find, name and write $\frac{1}{3}$, $\frac{2}{4}$ and $\frac{3}{4}$.	For $\frac{1}{2}$ and $\frac{1}{4}$ see year 1 For $\frac{3}{4}$ children must identify three groups. 	For $\frac{1}{2}$ and $\frac{1}{4}$ see year 1 Children to draw dots and divide followed by identifying three groups. 	$\frac{3}{4}$ of 12 is 3 Children will begin to divide by the denominator and multiply by the numerator $12 \div 4 = 3$ $3 \times 3 = 9$
Recognise the equivalence of $\frac{1}{2}$ and $\frac{2}{4}$	Cut an object or groups of objects into halves and quarters to physically compare.  	 one-half  two-quarters	$\frac{1}{2} = \frac{2}{4}$ Which is bigger, $\frac{1}{2}$ of 8 or $\frac{2}{4}$ of 8?

Fractions: Years 3 and 4

Objective & Strategy	Concrete	Pictorial	Abstract
Count up and down in tenths/hundredths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10	Use counting stick and base 10 to show a whole in ten equal pieces  10 pennies in a 10p can help this.		0.1, 0.2, 0.3, 0.4... $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$
Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators	Dividing objects into groups and counting amount in each group. 		What is $\frac{3}{4}$ of 12? $12 \div 4 = \boxed{}$ $3 \times \boxed{} = \boxed{}$ $\frac{3}{4} \times 12 = \boxed{}$
Recognise and show, using diagrams, equivalent fractions with small denominators and from families of common equivalent fractions	As year 2	"	"
Add and subtract fractions with the same denominator	Use fraction mats.  5 tenths	$\frac{1}{5} + \frac{3}{5}$ 	$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$ Add the numerator and leave the denominators the same.

Fractions: Years 5 and 6

Objective & Strategy	Concrete	Pictorial	Abstract
Compare and order fractions whose denominators are all multiples of the same number	Fraction tiles to physically overlay or match fractions		$\frac{3}{4} > \frac{4}{8}$
Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths	Same as Years 3 & 4	"	"
Add and subtract fractions with denominators that are multiples of the same number	Use fraction tiles 	$1\frac{7}{8} + 2\frac{1}{4} =$ 	$\frac{1}{2} + \frac{1}{3} = ?$ Make the denominators the same $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ $\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$
Use common factors to simplify fractions; use common multiples to express fractions in the same denomination	N/A	N/A	$\frac{4}{28} \div 4 = \frac{1}{8}$ Both denominator and numerator are multiples of 4
Compare and order fractions, including fractions > 1	 Use fraction tiles		$1\frac{4}{6} > 1\frac{1}{3}$